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ABSTRACT

Technologies play a prominent role in current reforms of school and collegiate mathematics. This study examined ways in which first-semester calculus students still showed the influence of a graphing-intensive college algebra course they had studied one or two semesters earlier. Students (n=18) at a large research university were asked to solve a series of problems in individual, audiotaped interviews during the last month of a traditionally taught calculus course. Two matched groups of students participated: experimental students who had studied college algebra in sections that integrated the use of graphing technologies and comparison students from traditionally taught sections of college algebra. The data showed that graphing students did continue to use technologies but mainly in routine rather than advanced ways. Main conclusions of the study were: (1) graphing technologies do have a lasting impact on students, even when the use of these tools is discouraged or prohibited, and (2) many students do not become sophisticated users nor do they appear to gain the expected lasting enhancements of conceptual knowledge during a one semester course. The implication of these findings is that careful attention to pedagogical issues must accompany curricular integration of technologies if changes are to significantly improve students' conceptual learning in mathematics. (Author/MKR)

Lasting Effects of the Integrated Use of Graphing Technologies in Precalculus Mathematics[1]

William O. Martin

Abstract

Technologies play a prominent role in current reforms of school and collegiate matheamtics. Studies over the past decade have shown that technologies can be used in ways that promote enhanced learning of baxic mathematical concepts such as function. However, there is little evidence about the robustness or durability of gains beyond an individual course. This study examined ways in which first-semester calculus students still showed the influence of a graphing-intensive college algebra course they had studied one or two semesters earlier. Eighteen students at a large research university were asked to solve a series of problems in individual, audiotaped interviews during the last month of a traditionally taught calculus course. Two matched groups of students participated: (a) experimental students who had studied college algebra in sections that integrated the use of graphing technologies and (b) comparison students from traditionally taught sections of college algebra. The data showed that graphing students did continue to use technologies but mainly in routine rather than advanced ways. Neither group of students showed strong preparation for calculus from the precalculus course. The main conclusions of this study were that (a) graphing technologies do have a lasting impact on students, even when the use of these tools is discouraged or prohibited, and (b) many students do not become sophisticated users nor do they appear to gain the expected lasting enhancements of conceptual knowledge during a one semester course. The implication of these findings is that careful attention to pedagogical issues must accompany curricular integration of technologies if changes are to significantly improve students' conceptual learning in mathematics.

What Impact Might Technologies Have on Learning and Retention?

Graphing calculators are increasingly being used in school and college mathematics classes. New ideas for uses of graphing technologies in mathematics classes are regularly featured at conferences and in the mathematics education literature; especially, in journals directed at school and college mathematics instructors (several recent examples include Curjel, 1993; Naraine, 1993; Demana & Waits, 1993). Curricular materials that draw on graphing technologies are also appearing; many textbooks acknowledge the existence of graphing technologies and several new textbooks for precalculus (e.g., Demana, Waits, & Clemens, 1992) and calculus (e.g., Dick & Patton, 1992) *require* student access to graphing tools. Is the movement toward increased use and integration of graphing technologies in mathematics, and particularly in precalculus, appropriate? Some research suggests that

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the answer is "yes" but the question needs much more study (Dunham, CITATION REQUIRED). Among the questions that could be raised about the integrated use of graphing technologies in precalculus are the following:

- How do students who have used graphing technologies while studying mathematics draw on this knowledge in subsequent mathematics courses?
- Do differences between groups with graphing and traditional backgrounds, as have been hypothesized and found by researchers, persist after the precalculus course?
- Do graphing approach students, because of decreased emphasis on traditional manipulative skills in algebra, appear disadvantaged during a subsequent traditionally taught mathematics course?

The use of technologies in school and college mathematics instruction has received considerable attention during the past decade. Technologies have been used in ways that promoted enhanced conceptual learning (Palmiter, 1991; Tufte, 1990) with little or no detrimental effect on students' abilities to carry out necessary procedures (Heid, 1988; Schrock, 1989). In precalculus courses, graphing technologies fostered student learning of important graphical concepts (Browning, 1988; Calculator and Computer Precalculus project [C^2PC] field test [1988-89], Harvey and others, unpublished) while promoting positive changes in attitudes and classroom interactions (Dunham, 1991; Farrell, 1990). Current views of learning and knowledge (see, for example, Hiebert & Carpenter, 1992) suggest that such changes should be long-lasting, but there is little research evidence to support the belief that these benefits are robust and enduring. This research study explored the lasting impact of the integrated uses of graphing technologies in college algebra. College students, who had previously used the Demana and Waits (1990) precalculus textbook in their college algebra course, were interviewed during the last month of their traditionally taught first semester calculus course. The problem-solving interviews were designed to generate data related to the above questions.

The decision to work with students who had studied precalculus with the C^2PC materials was made for several reaons. A variety of studies, including a large-scale field test in 1988-89, had shown cognitive and affective benefits for high school and college students who had used versions of the materials with computer graphing software and handheld graphing calculators, so it was reasonable to expect that the use of this approach would have similar benefits for the participants in this study. Students who participated in the study had taken college algebra in many different sections and with a variety of instructors, none of whom were especially committed to reform efforts or the use of technologies. The department of mathematics at this institution is very concerned about the role of precalculus couses as preparation for calculus. There is a faculty coordinator assisted by a graduate student, common tests and grading curves were used in traditionally taught sections, and a syllabus is prepared each semester to help ensure uniform coverage. The deparment makes strict use of placement test scores for initial placement in mathematics courses. The experimental sections did not share the common syllabus and tests, but they were also designed to feed students into the regular calculus sequence and so shared many of the goals of the regular sections of college algebra.

Because several earlier studies had shown gains in conceptual knowledge or higher levels of graphical reasoning for students who regularly used technologies I expected to find several positive, lasting benefits of the graphical approach to precalculus. Conceptual knowledge is seen a richly linked (Hiebert & Carpenter, 1992); such connections should help students use ideas in new ways and help them retain the knowledge longer. Specifically, this study was designed to test five hypotheses:

1. *The problem solving and mathematics skills that students gain in a graphing approach precalculus course persist into calculus.*

2. *Because graphing students draw on graphical perspectives in new situations, they better develop important concepts in calculus, such as limits and derivatives, than do students with a traditional precalculus background.*

3. *Students with a graphing background successfully develop, use, and integrate information from multiple representational forms. These students show a richer, more fully integrated functional concept than do students with a traditional precalculus background.*

4. *Graphing approach students show greater flexibility than do traditional background students when faced with problems they cannot immediately solve.*

5. *Graphing approach students are well prepared for and successful in calculus. Their readiness and preparation compares favorably with that of traditional background students.*

Technologies provide tools that can increase the cognitive power of students by (a) reducing or eliminating the need for extensive development of routine manipulative skills before concepts can be studied, (b) providing multiple perspectives on important concepts that will help students to construct their mathematical knowledge, and (c) promoting a broader, more unified view of basic mathematical notions, particularly *function*, that can serve as a common thread running through courses (Philipp, Martin, & Richgels, 1993). It was not expected that there would be serious detrimental effects for graphing background students even in a traditionally taught calculus setting; it seemed that the graphing students' enhanced conceptual knowledge could help them to develop necessary skills as needed.

Related Research

Four areas of existing mathematics education research provided a context for the study: (a) research on students learning to understand graphical representations of functions; (b) the influence of technologies on the balance between concepts and procedures in mathematics courses; (c) studies that investigated learning in courses using graphing utilities; and (d) the apparent impact of precalculus on subsequent mathematics course work. The fourth area, the focus of this study, contained almost no published research.

Research and theoretical discussion relating to students' understanding of functions and graphs is widespread; one review including scores of studies was given by Leinhardt, Zaslavsky, and Stein (1990). Much of this shows that students have serious conceptual difficulties in the domain (e.g., Goldenberg, 1988); much less research shows how to remedy the situation (but see, as an example, Mokros & Tinker, 1987). More positively, several researchers have found that technologies can be used to enhance learning in precalculus and calculus classes (Beckmann, 1989; Dugdale, 1990; Heid, 1988; Schrock, 1989; Tufte, 1990). Studies suggested that students in the experimental courses performed just as well as traditional groups on routine computational tasks (C²PC field test data, Harvey, et al., unpublished; Heid, 1988; Judson, 1988). Only one study looked for transfer of skills from graphing precalculus to a graphing unit in a subsequent physics course; in that study, Nichols (1992) detected no transfer of graphing skills from the prior use of graphing technologies in precalculus mathematics to the use of graphs in introductory physics. Dunham (REFERENCE) and Dunham and Dick (REFERENCE) compiled more recent summaries of research in this area.

Design of the Study

Participants

Between 1987 and 1992 the Department of Mathematics at the University of Wisconsin offered several sections of graphing approach precalculus (college algebra, trigonometry) along with many regular, traditionally taught precalculus sections. About 130 students who studied college algebra in graphing approach sections between 1990 and 1992 provided a pool of potential experimental subjects for the study. Nine of these students, who were still enrolled in a first-semester, traditionally taught calculus course after eight weeks of the Fall 1992 semester, were paired with nine comparison students from the same calculus discussion sections. The comparison students had traditional college algebra backgrounds at the University of Wisconsin-Madison. The students were matched as closely as possible based on their grade for college algebra, cumulative grade point average, high school mathematics units, and gender. Data for the study came from individual, audiotape recorded, "thinking aloud" sessions; university records; and questionnaire responses from the 18 students and their calculus instructors.

Data

The most important data were the students' work on nine problems drawn from precalculus and calculus. The free-response problems ranged from routine precalculus (solve a quadratic system of two equations in two unknowns) to conceptual calculus (sketch a graph for a function based on a sign chart for the function and its first two derivatives). Graphing technologies could have contributed useful information for several problems although they did not provide an obvious advantage. Other problems involved graphical concepts, but did not offer any opportunity to use a graphing utility. Several of the problems are included as illustrations of the differing nature of tasks

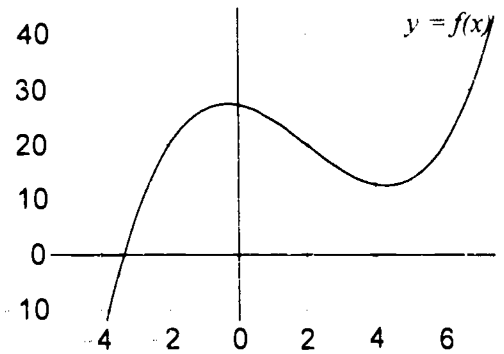
For each of the following, decide whether or not the limit exists. If not, give a reason. If so, find the value of the limit.

(a) $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$ *(b)* $\lim_{x \to \infty} \frac{x^3 - 8}{x^2 - 4}$ *(c)* $\lim_{x \to -2} \frac{x^3 - 8}{x^2 - 4}$

(d) *Carefully sketch a complete graph for the function* $f(x) = \frac{x^3 - 8}{x^2 - 4}$.

Figure 1 Problem 5

used in the study. Figure 1 contains Problem 5, drawn from beginning differential calculus.[2] It did not require the use of technologies but graphing tools could have been useful in the search for answers to each of its four parts. Figure 2 contains Problem 1; this warmup problem involved graphical concepts without any possibility of using a graphing utility to obtain a solution.[3]



Here is the graph of a function $y = f(x)$. Find a value of c so that $f(x) = c$ has three real solutions.

Figure 2 Problem 1

Data Analysis

Two characteristics of the participants' work were of interest: (a) the *outcome*, or degree of success achieved on problems; and (b) the *processes* by which students attempted to solve the problems. I used several coding schemes to record this information; the codes were used to help identify patterns in the written and spoken data. The outcome codes rated the degree of success achieved on the problem on a 0-5 scale while the process codes were used to generate frequencies of various solution strategies and behaviors. During the coding process I also wrote extensive notes about the students' work. Several statistical techniques, including factor and cluster analysis, were used to explore the data. In cases where there were noticeable differences between the two groups I used the nonparametric Wilcoxon matched-pairs signed-ranks test; I used a significance level of $p < 0.05$ for statistical tests in the study.

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[2] It turned out that Problem 5 produced the greatest variability of solution strategies and levels of success and, thus, some of the most useful data developed by the study.

[3] Problem 1 was intended as a gentle warmup exercise to help the students become accustomed to "thinking aloud." Surprisingly, more than half of students were unable to solve it correctly.

Results and Conclusions

Results

The data showed that there were statistically significant differences ($p < 0.05$) between the groups in the ways that students approached problems and in their uses of graphing technologies. Graphing background students not only made more use of graphing technologies and strategies as they worked on the problems; they also reported greater use of technologies in their traditionally taught calculus course. Two traditional background students reported slight prior exposure to graphing technologies and used graphing technologies in calculus, but the graphing students reported using technologies in a much wider range of ways. The extent to which students used technologies was significant because students reported either indifference to, or even outright prohibition of, the use of technologies in their traditionally taught calculus courses. Several graphing students made remarks indicating that they believed they should avoid dependence on technologies if they were to be successful in regular mathematics courses.

There were no significant differences between the groups in their readiness for calculus (their own and their instructors' impressions), final calculus grades, or degree of success on most of the research problems. Signs of differences reported by earlier studies could be seen in the pattern of results, but all but one of the differences were not statistically significant.[4] The one significant difference in success rates ($p < 0.05$) was on Problem 1 (six graphing background and two traditional background students correctly solved the problem): A similar pattern, but with smaller differences between groups, had been observed on a similar multiple-choice problem administered to all college algebra students at the University of Wisconsin-Madison during 1991-92.

The largest differences found by the study were in comparisons of success rates on different problems instead of comparisons between the two groups of students. It was striking that, aside from the mentioned differences, the groups were so similarly successful with routine, precalculus tasks and had little or no success with more conceptual tasks drawn from calculus. Table I provides some information about the students' performance on the problems and reveals some of the patterns of success on different tasks.

None of the graphing approach precalculus students could be characterized as *advanced* users of graphing technologies. Instead, they used the graphing calculators in *routine* ways such as to find the intersection of two functions. They had little success using technologies to gain insights to problems that they could not solve. For example, even though some students wanted to factor the numerator and denominator of the function in Problem 5, they did not think to use the graphing calculator to assist when they were uncertain about the factoring identities. Neither did they use technologies to help with an optimization problem that was completely solved by none of the 18 students.

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[4] For example, the graphing students were more successful with all tasks that required graph interpretation. They also had greater success solving an inequality, a problem that could be solved graphically or algebraically.

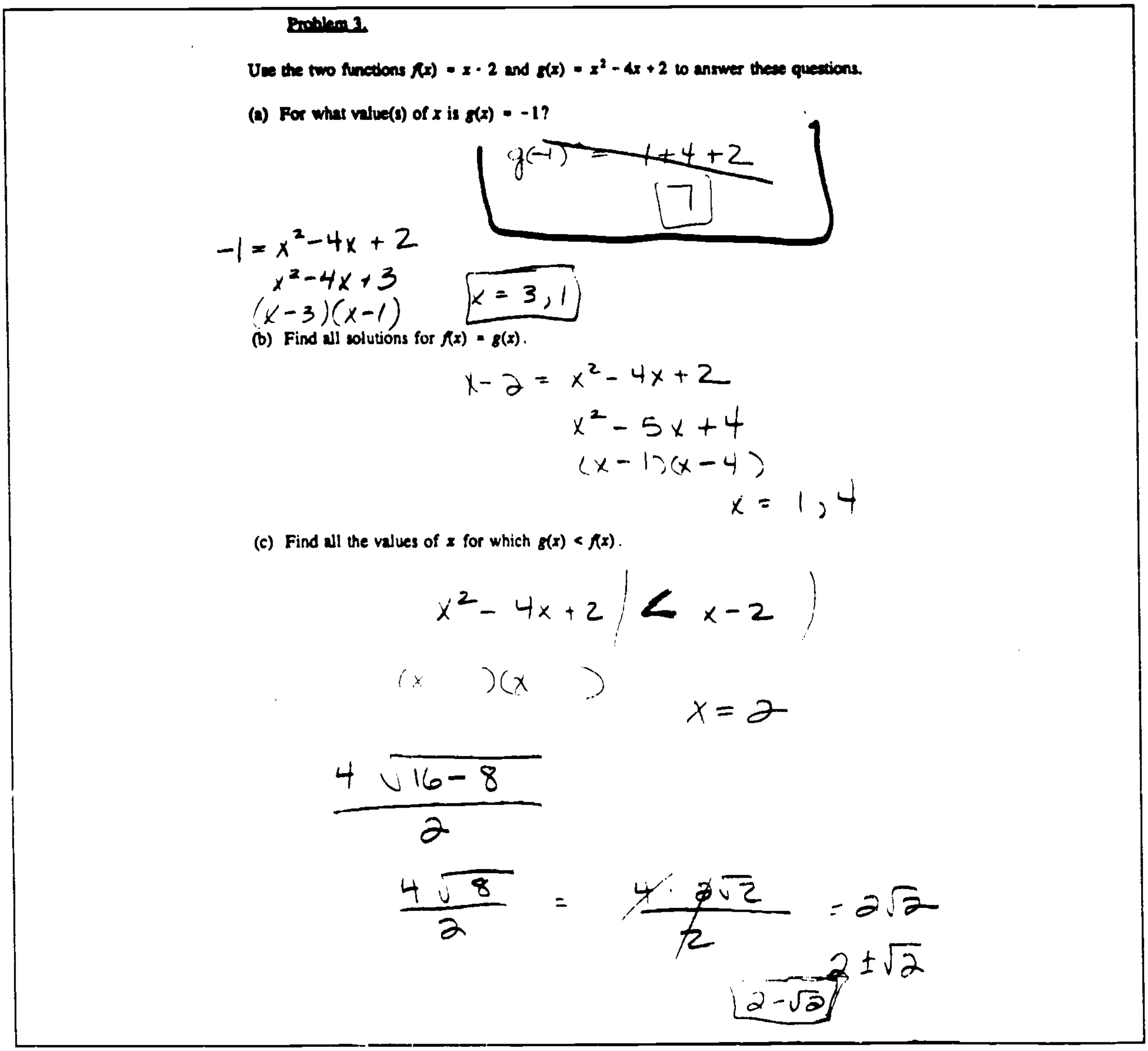
Table 1 Test and Subtest Scores for Degree of Success Achieved on Problems

Subtest	Mean Score[a]		Wilcoxon n = 9 pairs		
	Graph	Trad	G > T	G < T	p =
			(mean rank)		
All Problems	3.18	3.11	5 (5.00)	4 (5.00)	0.7671
Precalculus	3.79	3.59	6 (5.00)	3 (5.00)	0.3743
Calculus	2.52	2.70	4 (4.25)	5 (5.60)	0.5147
Graph Interpretation Necessary	3.43	3.21	6 (4.92)	3 (5.17)	0.4069
Algebraic Manipulation Necessary	2.83	3.08	4 (3.50)	4 (5.50)	0.5754
Choice of Algebraic or Graphical Strategy	3.07	2.97	4 (6.25)	5 (4.00)	0.7671
Graphing Calculator Possible	2.97	2.80	5 (4.80)	3 (4.00)	0.4008

[a] The scores reported in this table are *mean score per problem* for each test or subtest. This facilitates comparisons between tests and interpretation of absolute levels of scores. The codes awarded for the students' degree of success on problems had the following interpretations:

5 = correct 4 = basically correct 3 = good progress 2 = little progress 1 = nothing relevant 0 = blank

Some more qualitative analyses of the data revealed interesting contrasts between the two groups of students that warrant further investigation. In an effort to develop an understanding of the students' problem solving strategies and thinking about mathematics I coded information about ways that they approached the problems, such as the kinds of strategies used, their reactions to impasses, and the types of representations they used. Naturally, I found considerable variability within each group. There also appeared several differences between the groups that, although not statistically significant (partly, at least, because of the small sample size), followed a pattern that could reflect expected patterns of thinking due to differences between graphing and traditional approaches. The most notable of these differences was in students' inclination to use *inappropriate standard strategies*. A simple example of this is the use of l'Hôpital's on problem 5(c); several students did that after appropriately using the rule on 5(a) and 5(b). It seemed that students used the rule because of the appearance of the problem and without checking that it actually applied in this situation. A more revealing illustration is given by the work of one traditional student, identified as T8, on the precalculus Problem 3 shown in Figure 3. A transcription of this students' comments while working on the problem is given in Figure 4.



Problem 3.

Use the two functions $f(x) = x - 2$ and $g(x) = x^2 - 4x + 2$ to answer these questions.

(a) For what value(s) of x is $g(x) = -1$?

$\cancel{g(-1) = 1 + 4 + 2}$ $\boxed{7}$

$-1 = x^2 - 4x + 2$

$x^2 - 4x + 3$

$(x-3)(x-1)$ $\boxed{x = 3, 1}$

(b) Find all solutions for $f(x) = g(x)$.

$x - 2 = x^2 - 4x + 2$

$x^2 - 5x + 4$

$(x-1)(x-4)$

$x = 1, 4$

(c) Find all the values of x for which $g(x) < f(x)$.

$x^2 - 4x + 2 < x - 2$

$(x \quad)(x \quad)$

$x = 2$

$\dfrac{4 \quad \sqrt{16-8}}{2}$

$\dfrac{4 \quad \sqrt{8}}{2} = \dfrac{4 \cdot 2\sqrt{2}}{2} = 2\sqrt{2}$

$2 \pm \sqrt{2}$

$\boxed{2 - \sqrt{2}}$

Figure 3 Work of traditional group student T8 on Problem 3

The student T8 seemed to solve problems by choosing a remembered technique that was based on the surface appearance of the problem. In solving parts (a) and (b) T8 was successful (after correcting an initial error in the first part) because the chosen technique was appropriate. The work on part (c), however, is unrelated to the content of the problem; instead, it applies standard manipulative techniques, sometimes carried out incorrectly, used to solve linear and quadratic equations. The remarks made while working on the problem indicate that T8 also considered, and rejected, using other techniques because the inequality symbol appeared in the problem. The use of standard manipulative procedures in an invalid context, such as illustrated by T8's work on Problem 3, provides important evidence of the student's beliefs about the nature of mathematical problem solving. If this only happens once we might attach no more significance to it than carelessness (especially in a case such as 5(c)). However, in this study several students, both from the graphing and the traditional groups, repeatedly used inappropriate standard strategies, suggesting an undesirable view of mathematical problem solving.

Problem 3 {Side 1: 192}

Um, I just believe you fill in negative one for g of x. So it equals seven I think.

[Reads part (b)] {Side 1: 206}

Just set the two functions equal to each other. And factor. And then x would equal one and four.

[Reads part (c)] {Side 1: 214}

Hm. Um, I'm not real sure about this one.

I: Remember to try to tell me what you're thinking as you work.

Okay. Let's see. I mean, usually when you use less than or greater than, you know what, when you divide by negative and the sign changes. But I don't know, that's not going to do any good in this problem. I don't think. Or you just solve for x. Um ... Either that or just solve each one individually. So x would be equal to two and this one ... you can't factor, you'd have to use ... negative, that, oh I can't think what it's called. Maybe, I can't, I don't know what the name of the formula was. Negative b plus or minus b squared minus four a c over two a.

I: Okay, I know what you mean.

Okay. Square ... [silence, writing sounds] So ... I think that's plus or minus, two plus or minus square root two, I think. So ... the values for x in which g of x is greater than, or less than f of x would be two minus the square root of two, I think. Would be the only one. Not that. I don't know.

I: Okay. Do you believe you've correctly solved each part of this problem? Again, go through each part.

Um, the first one I think so. Second one, for sure. And third one ... possibly. [laughs]

I: You're not so sure on the third one. Okay.

Right.

Figure 4. Transcription of the comments T8 made while working on Problem 3

Traditional background students, including T8, used inappropriate standard procedures half again as often (36 times) as did graphing background students (25 instances). Four of the nine traditional students each used inappropriate strategies at least four times, accounting for 25 of the 36 cases; only two graphing students used inappropriate strategies at least four times for a total of 9 instances. One might expect that weaker students would be most inclined to apply procedures inappropriately but this was not the case. Several of the students who used many inappropriate strategies did have lower grades for college algebra and calculus (C's), but this group also included students who did quite well in both courses: One of these two graphing students earned an AB in calculus and had a BC for college algebra; two of the aforementioned traditional students had AB's and B's for the two courses. (T8 had a BC for calculus and a B for college algebra).[5]

This pattern, although not convincing, fits with ideas about differences between traditional and graphing approaches in precalculus. Were similar differences found to hold in further studies, it could reflect that many students in traditional precalculus courses learn to recognize problem types and apply specific techniques while students from graphing approach courses focus more on understanding problem situations from a variety of perspectives. When students with these backgrounds face an unfamiliar problem, a traditional background student may be more likely to try to impose what seems like an appropriate solution strategy from his or her repertoire. Graphing students, on the other hand, might be more inclined to explore the problem, perhaps using technologies and various representations. If true, this would indicate that the use of technologies can, in fact, contribute to the development more powerful and flexible problem solving capabilities as hypothesized. The results of this study hint that this could be the case, but are not strong enough to offer confirmation.

Conclusions

Graphing students in this study did not display the hypothesized enhanced conceptual knowledge of important calculus concepts. The graphing precalculus course appeared to have had a lasting influence, but not to the extent nor in the conceptual domain that one might have hoped for. Graphing students did not appear disadvantaged in the traditional calculus course and they continued to draw on a graphical perspective in their work even though this was not encouraged by their instructors. In a search for possible explanations of these findings I considered three groups of issues that could account for the weaker than expected results: (a) issues surrounding the design of the study, (b) issues relating to the mathematics courses that were studied, and (c) issues relating to the broader educational context of the study and courses.

Several noticeable differences between the groups were in the hypothesized direction but were not found to be statistically significant. One problem was that the group of students enrolled in calculus provided a sample that was considerably smaller than had been anticipated during the planning stage. This reduced the power of the statistical tests used to detect significant differences between the groups. For example, graphing students were noticeably less inclined to unquestioningly apply inappropriate strategies. Similarly, graphing students had more success with graph interpretation tasks, although not significantly so. Perhaps true differences, in the direction one would predict, were not detected because of the lack of statistical power.

I do not believe that the lack of statistical power accounts for the small number of significant differences detected between the two groups. Instead, the extensive information obtained about each student by this study suggests that both groups were, in fact, highly comparable. The impression I gained from the data and my contact with the students during the interview sessions was that neither group had the sort of strong, conceptual mathematical knowledge applicable in new or different situations that one would hope for at this level.

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[5] One interesting sidelight to the findings from this study was the lack of any apparent relationship between performance on these problems and grades in precalculus or calculus.

Why, then, had the graphical precalculus not developed the conceptual power that I had expected? I believe that (a) the nature of the precalculus and calculus courses and (b) their position in the broader educational context accounted for the pattern of results found by the study. Dugdale's work (1990) emphasized that the way in which technologies are used is at least as important as whether or not they are used. Although the Demana and Waits textbook (1990) emphasizes the use of technologies to develop conceptual knowledge of functions, their textbook was used in a very traditional way by the students involved in this study. That is, instructors would explain new material and the students would work on similar problems. This approach would not give students the important experiences with independent work in unfamiliar problem situations. In contrast, students who study mathematics in more radically restructured courses, such as the calculus courses studied by Crocker (1991), may show greater lasting differences in their approach to mathematics because technology is used to restructure the course, not just the content.

I also question how much a single course can be expected to change students with at least 12 years' of experience in predominantly, if not entirely, traditional courses. Along the same lines, to what extent do the motivations that students bring to mathematics courses at this level moderate the impact of new approaches? Many students who take precalculus mathematics in college seem more concerned with grades and credits than with developing mathematical knowledge. This, too, would act to limit the lasting impact of any course.

The main conclusions of this study were that (a) graphing technologies do have a lasting impact on students, even when the use of these tools is discouraged or prohibited, and (b) many students do not become sc histicated users nor do they gain lasting enhancements of conceptual knowledge during a one semes r course delivered in a fairly traditional manner. I view the students' inclination to continue using technologies, even without encouragement, as a positive sign that technologies can be used to favorably influence their learning and attitudes toward mathematics. Students need to develop much better mathematical knowledge than demonstrated by the participants in this study. I believe that technologies do offer many potential benefits that could help to improve conceptual learning and mathematical power that lasts into subsequent courses. For this to happen, technologies must be used in more varied ways so that the benefits, such as found by earlier studies, persist.

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